

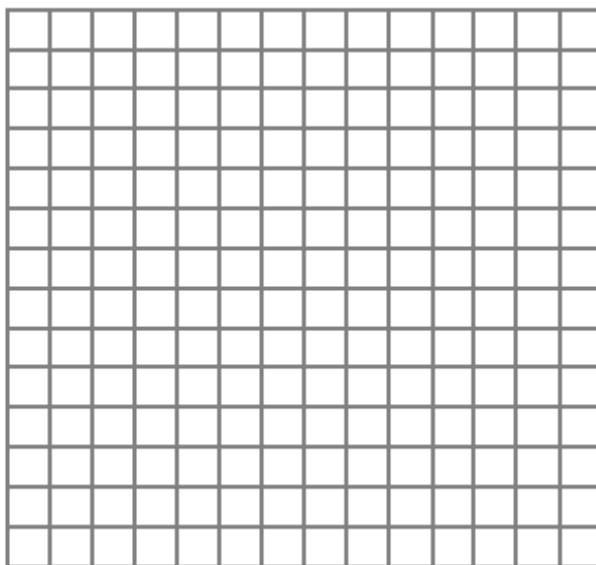
Name: _____

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A2CC: Logarithmic Functions

One of the properties of the exponential function $f(x) = b^x$ is that it is a 1-1 function. Remember that this means it has an inverse function whose graph can be obtained by reflecting the graph of $y = b^x$ through the line $y = x$.

Let's graph the inverse of the function that we were looking at yesterday, $y = 2^x$.



Write an equation for the inverse of $y = 2^x$.

The equation $x = b^y$ tells us that y is the exponent on b that produces x . In situations like this the word logarithm is used in place of exponent. A **logarithm** is an exponent. We can abbreviate to:

$$y = \log_b x$$

read as “ y equals log x to the base b ” or “ y equals log x base b .”

Exponential form: $x = b^y$

Logarithmic form: $y = \log_b x$

General Graphs of Logarithmic Functions

$$y = \log_b x, b > 1$$

$$y = \log_b x, 0 < b < 1$$

Properties of Logarithmic Functions

1. The domain consists of all positive numbers.
2. The range consists of all real numbers.
3. The function is increasing (the curve is rising) when $b > 1$, and its is decreasing (the curve is falling) when $0 < b < 1$.
4. It is a one-to-one function.
5. The point $(1,0)$ is on the curve.
6. There is no y-intercept.
7. The y-axis is a vertical asymptote to the curve.

Practice

1. Evaluate the following logarithms. If needed, write an equivalent exponential equation. (Without the use of your calculator.)

(a) $\log_2 8$ (b) $\log_4 16$ (c) $\log_5 625$ (d) $\log_{10} 100,000$

(e) $\log_6 \left(\frac{1}{36} \right)$ (f) $\log_2 \left(\frac{1}{16} \right)$ (g) $\log_5 \sqrt{5}$ (h) $\log_3 \sqrt[5]{9}$

It is critically important to understand that logarithms **give exponents as their outputs**. We will be working for multiple lessons on logarithms and a basic understanding of their inputs and outputs is critical.

2. If the function $y = \log_2(x+8)+9$ was graphed in the coordinate plane, which of the following would represent its y-intercept?

- (1) 12
- (2) 13
- (3) 8
- (4) 9

3. Between which two consecutive integers must $\log_3 40$ lie?

- (1) 1 and 2
- (2) 2 and 3
- (3) 3 and 4
- (4) 4 and 5

Calculator Use and Logarithms – Most calculators only have two logarithms that they can evaluate directly. One of them, $\log_{10} x$, is so common that it is actually called the **common log** and typically is written without the base 10.

$$\log x = \log_{10} x \quad (\text{The Common Log})$$

4. Evaluate each of the following using your calculator.

- (a) $\log 100$
- (b) $\log\left(\frac{1}{1000}\right)$
- (c) $\log \sqrt{10}$

5. Can the value of $\log_2(-4)$ be found? What about the value of $\log_2 0$? Why or why not? What does this tell you about the domain of $\log_b x$?

COMMON CORE ALGEBRA II HOMEWORK

FLUENCY

- Which of the following is equivalent to $y = \log_7 x$?
(1) $y = x^7$ (3) $x = 7^y$
(2) $x = y^7$ (4) $y = x^{\frac{1}{7}}$
- If the graph of $y = 6^x$ is reflected across the line $y = x$ then the resulting curve has an equation of
(1) $y = -6^x$ (3) $x = \log_6 y$
(2) $y = \log_6 x$ (4) $x = y^6$
- The value of $\log_5 167$ is closest to which of the following? Hint – guess and check the answers.
(1) 2.67 (3) 4.58
(2) 1.98 (4) 3.18
- Which of the following represents the y-intercept of the function $y = \log(x+1000) - 8$?
(1) -8 (3) 3
(2) -5 (4) 5
- Determine the value for each of the following logarithms. (Easy)
(a) $\log_2 32$ (b) $\log_7 49$ (c) $\log_3 6561$ (d) $\log_4 1024$
- Determine the value for each of the following logarithms. (Medium)
(a) $\log_2 \left(\frac{1}{64}\right)$ (b) $\log_3 (1)$ (c) $\log_5 \left(\frac{1}{25}\right)$ (d) $\log_7 \left(\frac{1}{343}\right)$

7. Determine the value for each of the following logarithms. Each of these will have non-integer, fractional answers. (Difficult)

(a) $\log_4 2$

(b) $\log_4 8$

(c) $\log_5 \sqrt[3]{5}$

(d) $\log_2 \sqrt[5]{4}$